Standard Deviations PS Solutions

1. The standard deviation of a set shows how much variation there is from the mean, how widespread a given set is. So, a low standard deviation indicates that the data points tend to be very close to the mean, whereas high standard deviation indicates that the data are spread out over a large range of values.

   Now, clearly set B={1, 2, 3, 4, 5, 6, 7, 8, 9, 10} is less widespread than set A, so its standard deviation is less than the standard deviation of set A;

   Set C={1, 3, 5, 7, 9, 11, 13, 15, 17, 19} is as widespread as set A, so its standard deviation equals to the standard deviation of set A (important property: **if we add or subtract a constant to each term in a set the standard deviation will not change**, since set A can be obtained by adding 9 to each term of set B, then the standard deviations of those sets are equal);

   Set D={2, 3, 5, 7, 11, 13, 17, 19, 23, 29} is more widespread than set A so its standard deviation is greater than the standard deviation of set A.

   Answer: C.

2. The standard deviation of a set does not change if a constant is added to all the members.

   Thus, standard deviation of (a,b,c) will be the same as of (a+ab, b+ab, c+ab).

   And, option E is the same as (a+ab, b+ab, c+ab).

3. Given: mean=21 and SD=6.

   The number of hours that Pat watched television was between 1 and 2 standard deviations below the mean: 1 SD below the mean is **mean-1*SD=15** and 2 SD below the mean is **mean-2*SD=9**, so the number of hours that Pat watched television was between 9 and 15 hours.

   Answer: D.

4. The mean of set S is $\frac{sum}{n}$, where n is the number of terms in set S.

   Since set S consist of positive numbers, then when we add -1 to the set the sum of the numbers in the new set will decrease. So, the new mean will be $\frac{less\ sum}{more\ terms}=\frac{less\ sum}{n+1}$, which will be less than $\frac{sum}{n}$. Hence the mean must decrease.

   Answer: B.

5. The standard deviation of a set shows how much variation there is from the mean, how widespread a given set is. So, a low standard deviation indicates that the data points tend to be very close to the mean, whereas high standard deviation indicates that the data are spread out over a large range of values.

   So when we add numbers, which are far from the mean we are stretching the set making SD bigger and when we add numbers which are close to the mean we are shrinking the set making SD smaller.

   According to the above **adding two numbers which are closest to the mean will shrink the set most**, thus decreasing SD by the greatest amount.

   Closest to the mean are 85 and 85 (actually these numbers equal to the mean) thus adding them will definitely shrink the set, thus decreasing SD most.

   Answer: D.

6. Mean is 4 and so are the means of all 5 pairs from answers choices.

   A. (-1, 9) These two numbers are farthest from the mean and they will stretch the set making SD bigger

   B. (4, 4) These two numbers are closest to the mean and the will shrink the set making SD smaller

   C. (3, 5) Suitable option so far

   D. (2, 6) Suitable option so far

   E. (0, 8) These two numbers are also far from mean and they will also stretch the set making SD bigger.

   So, when I looked at the options C and D I assumed that C is also too close to the mean and it will affect it more than D. So I ended with D and was correct. But still my logic eliminating C was not sure thing, without the calculations.

7. Set A - {2, 4, ..., 100};
   Set X - {-48, -46, ..., 50};
   Set Y - {3, 6, ..., 150};
   Set Z - {-2/4, -4/4, ..., -100/4} = {-1/2, -1, -3/2, ..., -25}.

   If we add or subtract a constant to each term in a set the SD will not change, so sets A and X will have the same SD.

   If we increase or decrease each term in a set by the same percent (multiply by a constant) the SD will increase or decrease by the same percent, so set Y will have 1.5 times greater SD than set A and set Z will have 4 times less SD than set A (note SD can not be negative so SD of Z wil be SD of A divided by 4 not by -4).

So, the ranking of SD's in descending order is: Y, A=X, Z.

Answer: D.

8. Say **Q={2, 3, 4}** (consider a simple set).
   The standard deviation (D) will be very small (~1).
   The mean (M) is 3;

   We want to add new element so that the standard deviation to decrease. Add the element which is equal to the mean. So, if N=3:

   D will decrease --> condition satisfied.
   N=M --> discard I and II.
   N>D --> discard III.

   Answer: E.

9. "How many of the 10 running times are more than one SD below the mean" means how many data points from given 10 are less than**mean-1SD**.

   We are given that SD=22.4, so we should find mean --> mean=100 --> there are only 2 data points below 100-22.4=77.6, namely 70 and 75.

   Answer: B.

10. Important note: SD is always >=0. SD is 0 only when the list contains all identical elements (or which is same only 1 element).

    So the answer is clearly E, none.

    Answer: E.

    To elaborate more: **Standard deviation shows how much variation there is from the mean, how widespread a given set is. So, a low standard deviation indicates that the data points tend to be very close to the mean, whereas high standard deviation indicates that the data are spread out over a large range of values.** So, basically we can say that it in a sense measures the distance and the distance can not be negative. Also if you look at the formula for the standard deviation you'll see that it's square root of some number (variance) and the square root can never be negative.

11.
    Let the smallest odd integer be 1, thus the largest one will be 5. We can have following 6 types of sets:

    1. {1, 1, 1, 5} --> mean=2 --> |mean-x|=(1, 1, 1, 3);

2. {1, 1, 3, 5} --> mean=2.5 --> |mean-x|=(1.5, 1.5, 0.5, 2.5);
3. {1, 1, 5, 5} --> mean=3 --> |mean-x|=(2, 2, 2, 2);
4. {1, 3, 3, 5} --> mean=3 --> |mean-x|=(2, 0, 0, 2);
5. {1, 3, 5, 5} --> mean=3.5 --> |mean-x|=(2.5, 0.5, 1.5, 1.5);
6. {1, 5, 5, 5} --> mean=4 --> |mean-x|=(3, 1, 1, 1).

CALCULATING STANDARD DEVIATION OF A SET {x1, x2, ... xn}:
1. Find the mean, m, of the values.
2. For each value x_i calculate its deviation ($m-x_i$) from the mean.
3. Calculate the squares of these deviations.
4. Find the mean of the squared deviations. This quantity is the variance.
5. Take the square root of the variance. The quantity is th SD.

Expressed by formula: $standard\ deviation = \sqrt{variance} = \sqrt{\frac{\sum(m-x_i)^2}{N}}$.

You can see that deviation from the mean for 2 pairs of the set is the same, which means that SD of 1 and 6 will be the same and SD of 2 and 5 also will be the same. So SD of such set can take only 4 values.

Answer: B.

12. Value is *more than 2.5SD* from the mean means that the distance between the mean and the value must be more than 2.5*SD=7.5. So the value must be either less than 20-7.5=**12.5** or more than 20+7.5=**27.5**.

    Answer: A.

13. 2.5 standard deviation equals to 2.5*2=5;

    2.5 standard deviations from the mean, so **5 points, from the mean is the range from {mean-5} to {mean+5}**, so from 15 to 25: (15, 25).

    The correct answer choice must cover all this range: only answer choice C (14.0; 26.5) does this.

    Answer: C.

14. **If we add or subtract a constant to each term in a set the standard deviation will not change.**

    Since each set can be obtained by adding some constant to each term of set X (20 for set A, 21 for set B and 12 for set C), then the standard deviations of all sets are the same.

    Answer: E.

15. TIP:
If we add or subtract a constant to each term in a set:
Mean will increase or decrease by the same constant.
SD will not change.

If we increase or decrease each term in a set by the same percent (multiply by a constant):
Mean will increase or decrease by the same percent.
SD will increase or decrease by the same percent.

So in our case SD won't change as we are adding 5 to each term in a set --> SD=d.

Answer: A.

16. The value which is **exactly** two SD less than the mean is: **mean-2*SD=13.5-2*1.5=10.5**.

Answer: A.

17. The standard deviation ({SD}) = 2;

3 standard deviations below the mean is greater than 43:

{Mean} - 3*{SD} > 43;
{Mean} - 6 > 43;
{Mean} > 49.

Answer: E.

18. **If we add or subtract a constant to each term in a set the standard deviation will not change.**

Set {r-2, s-2, t-2} is obtained by subtracting 2 from each term of {r, s, t}.
Set {0, r-s, t-s} is obtained by subtracting s from each term of {r, s, t}.
Set {r-4, s+5, t-1} is totally different from {r, s, t}.

Thus the answer is I and II only.

Answer: C.

19. Logically, SD of a set will decrease if you add numbers which are equal to its mean. Thus the answer should be E.

I will however provide you with mathematical reasoning to justify the above statement.

S.D. for hundred numbers = d = $\sqrt{\frac{S}{100}}$ where 'S' is sum of the squares of the difference between each number and the mean.

Now let the two numbers added be 'x' and 'y'.

S.D after adding the two numbers will be = $\sqrt{\frac{S}{102}+\frac{(x-6)^2+(y-6)^2}{102}}$

Now it is obvious that $\frac{S}{102}$ will be less than $\frac{S}{100}$. Also, the minimum value of $\frac{(x-6)^2+(y-6)^2}{102}$ will be 0 when both 'x' and 'y' are equal to 6.

Thus if the two numbers added are equal to the mean, the SD of the set must decrease. (Unless of course SD of the set was 0 to start with (not in our case) and then in that case SD will remain constant).

Answer : E

20. One standard deviation of the mean is from *{mean}-{deviation}=60-3.5=56.5* to *{mean}+{deviation}=60+3.5=63.5*. The *smallest* number within this range from the options is 59.

    Answer: C.

21. "Standard deviation shows how much variation there is from the mean. A low standard deviation indicates that the data points tend to be very close to the mean, whereas high standard deviation indicates that the data are spread out over a large range of values."

    So when we add numbers, which are far from the mean we are stretching the set making SD bigger and when we add numbers which are close to the mean we are shrinking the set making SD smaller.

    According to the above adding two numbers which are closest to the mean will shrink the set most, thus decreasing SD by the greatest amount.

    Closest to the mean are 6 and 6 (actually these numbers equal to the mean) thus adding them will definitely shrink the set, thus decreasing SD.

    Answer: E.

22. If 70% of population is within 1 standard deviation mean then 30% are more than 1 standard deviation below the mean.

    Taking symmetric about the mean,
    75 are more than 1 sd below the mean therefore these 75 constitute 15% (Taking 15% below the mean and 15% above the mean)

    Therefore total population is 500.

    Therefore 70% of 500 is 350. who are within 1 standard deviation from the mean. Answer B.

23. The standard deviation of a set shows how much variation there is from the mean, how widespread a given set is. So, a low standard deviation indicates that the data points tend to be very close to the mean, whereas high standard deviation indicates that the data are spread out over a large range of values.

    Set B is the least widespread, thus it has the least SD.

    Answer: B.

24. The value which is exactly two SD below the mean is: mean-2*SD=13.5-2*1.5=10.5.

    Answer: A.

25. "Standard deviation shows how much variation there is from the mean. A low standard deviation indicates that the data points tend to be very close to the mean, whereas high standard deviation indicates that the data are spread out over a large range of values."

    So when we add numbers, which are far from the mean we are stretching the set making SD bigger and when we add numbers which are close to the mean we are shrinking the set making SD smaller.

    According to the above adding two numbers which are closest to the mean will shrink the set most, thus decreasing SD by the greatest amount.

    Closest to the mean are 10 and 10 (actually these numbers equal to the mean) thus adding them will shrink the set most, thus decreasing SD by the greatest amount.

    Answer: E.

26. 2 standard deviations from the average is from *mean - 2*SD* to *mean + 2*SD*, thus from 50-2*50.5=51 to 50+2*50.5=151:

    -51 < m < 151
    -51 < n < 151

    -102 < m+n < 302.

    Only option D is in this range.

    Answer: D.

27. "Within 1.5 standard deviation of the mean" - means in the range **{mean-1.5*sd; mean+1.5*sd} = {8.1-1.5*0.3; 8.1+1.5*0.3} = {7.65; 8.55}**.

    From the 12 listed amounts, only one (7.51) is out of this range and 11 are within this range.

    Answer: E.

28. Mean increasing by certain % doesnt mean that median should increase by same %.

    Median will only increase by same % if all data point move ahead by same % but increase of mean does not guarantee that all data points are moving by same % as well. Answer is E.

    Consider the following set: {1, 2, 3} --> mean=median=2 and sum=6. Now, if we increase the mean by 100%, we increase the sum by 100%, so it'll become 12. But new set can be {0, 0, 12}: the third element increased and the first two elements decreased or {2, 2, 8}: the first and the third elements increased and the second remained the same...

    You can apply the same logic to the question at hand.

    Answer: E.

29. Set A - {11, 13, 17, 19, 23};
    Set B - {5 consecutive even integers}, for example - {12, 14, 16, 18, 20};
    Set C - {5 consecutive multiples of 7}, for example - {7, 14, 21, 28, 35}.

    Now, the standard deviation of a set shows how much variation there is from the mean, *how widespread a given set is*. So, a low standard deviation indicates that the data points tend to be very close to the mean, whereas high standard deviation indicates that the data are spread out over a large range of values.

    You can see that set C is most widespread and set B is least widespread, so the correct answer is: B, A, C.

    Answer: E.

30. 31.0 is more than 2.5 standard deviations from 24 --> 31 > 24 + 2.5*{SD} --> 2.5*{SD} < 7 --> {SD} < 2.8. Only option E offers the standard deviation less than 2.8.

    Answer: E.

31. A score of 58 was 2 standard deviations below the mean --> 58 = Mean - 2d
    A score of 98 was 3 standard deviations above the mean --> 98 = Mean + 3d

    Solving above for Mean = 74.

    Answer: A.

32. "Within 1 standard deviation of the mean" - means in the range {mean-1*sd; mean+1*sd} = {10-*0.3; 10+0.3} = {9.7; 10.3}.

    From the 8 listed numbers, 6 are within this range so 6/8=75%.

    Answer: D.

33. "How many of the 10 running times are more than one SD below the mean" means how many data points from given 10 are less than**mean-1SD**.

    We are given that SD=22.4, so we should find mean --> mean=100 --> there are only 2 data points below 100-22.4=77.6, namely 70 and 75.

    Answer: B.